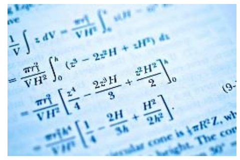
AP Calculus BC



COVER PAGE
SUMMER WORK

School Year - 2019-2020

NAME: ___________________________________

PLEASE READ ALL STATEMENTS BELOW!!!!

Put your name on the line above.

This is the first page of your summer work.

Write neatly in the spaces given below each problem.

Show all work for each problem. No work = No credit

You must STILL recall the values from the unit circle and practice unit circle. You must also know the tangent values associated with the functions of the unit circle.

This work is due in its entirety on the first day of class in August – NO EXCEPTIONS!!

All supplies must be ready for class on the first day.

This counts as 169 points of the first marking period grade.
It is in your best interest to complete all of the problems to the best of your ability!!

Huebner
A.I. DuPont High School
2019-2020

Part I: First, let's whet your appetite with a little Precalc! (26 points)

4 1) For what value of k are the two lines $2x + ky = 3$ and $x + y = 1$

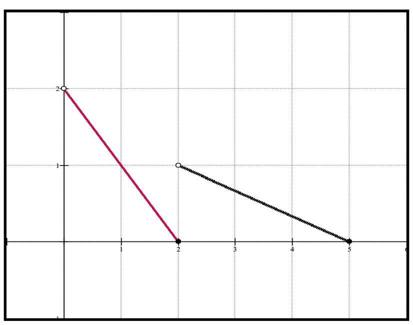
(a) parallel? (b) perpendicular?

2 2) Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at the point (3, 4) in slope intercept form.

5 3) Graph the function shown below. Also indicate any key points and state the domain and range.

$$f(x) = \begin{cases} 4 - x^2, & x < 1 \\ \frac{3}{2}x + \frac{3}{2}, & 1 \le x \le 3 \\ x + 3, & x > 3 \end{cases}$$

4 4) Write a piecewise formula for the function shown. Include the domain of each piece!



5 5) Graph the function $y = 3e^{-x} - 2$ and indicate asymptote(s). State its domain, range, and intercepts.

For #6-7, parametric equations are given. Complete the table and sketch the curve represented by the parametric equations (label the initial and terminal points as well as indicate the direction of the curve).

3 6) $x = 4\sin t, \quad y = 2\cos t, \quad 0 \le t \le 2\pi$

t	**0**	$\pi/4$	$\pi/2$	$3\pi/4$	π	$3\pi/2$	2π
x							
y							

3 7) $x = 2t - 5, \quad y = 4t - 7, \quad -2 \le t \le 3$

t	**−2**	**−1**	**0**	**1**	**2**	**3**
x						
y						

Part II: Unlimited and Continuous! (21 points)

For #1-4 below, find the limits, if they exist.(#1-13 are 1 pt each)

1) $\lim_{x \to 4} \frac{2x^3 - 7x^2 - 4x}{x - 4}$

2) $\lim_{x \to 9} \frac{\sqrt{x} - 3}{9 - x}$

3) $\lim_{x \to 1} \frac{x^2 - 2x - 5}{x + 1}$

4) $\lim_{x \to -2} \frac{x^3 + 8}{x + 2}$

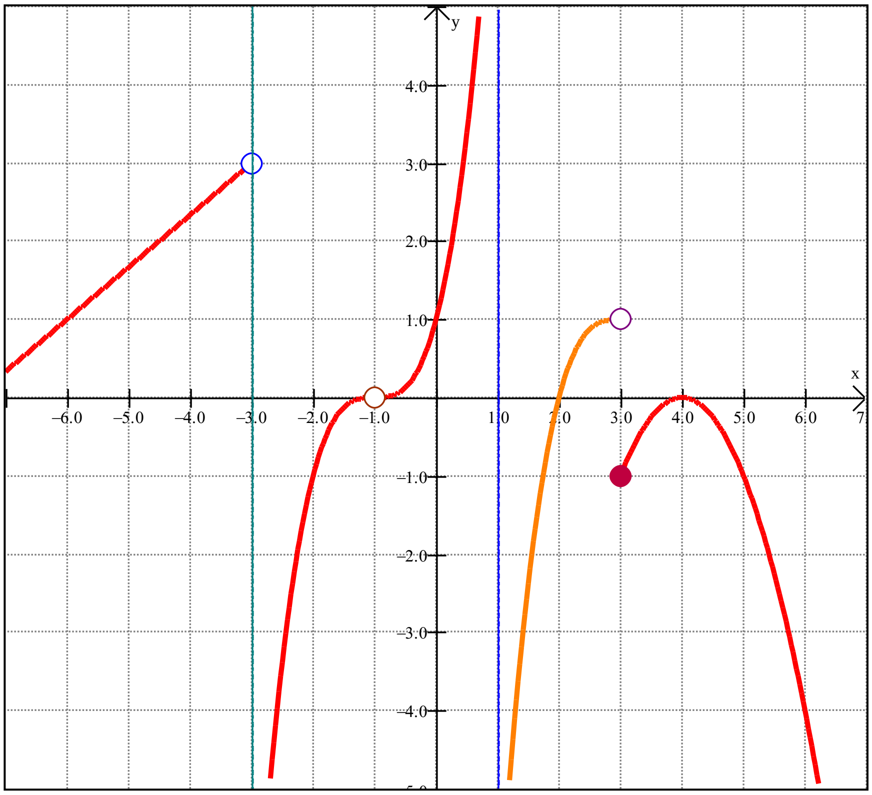
For #5-7, explain why each function is discontinuous and determine if the discontinuity is removable or nonremovable.

5) $g(x) = \begin{cases} 2x - 3, & x < 3 \\ -x + 5, & x \ge 3 \end{cases}$

6) $b(x) = \frac{x(3x + 1)}{3x^2 - 5x - 2}$

7) $h(x) = \frac{\sqrt{x^2 - 10x + 25}}{x - 5}$

For #8-13, determine if the following limits exist, based on the graph below of p(x). If the limits exist, state their value. Note that x = -3 and x = 1 are vertical asymptotes.



8) $\lim_{x\to 1^-} p(x)$ 9) $\lim_{x\to -3^-} p(x)$ 10) $\lim_{x\to 2} p(x)$

11) $\lim_{x\to 3^-} p(x)$ 12) $\lim_{x\to 3^+} p(x)$ 13) $\lim_{x\to -1} p(x)$

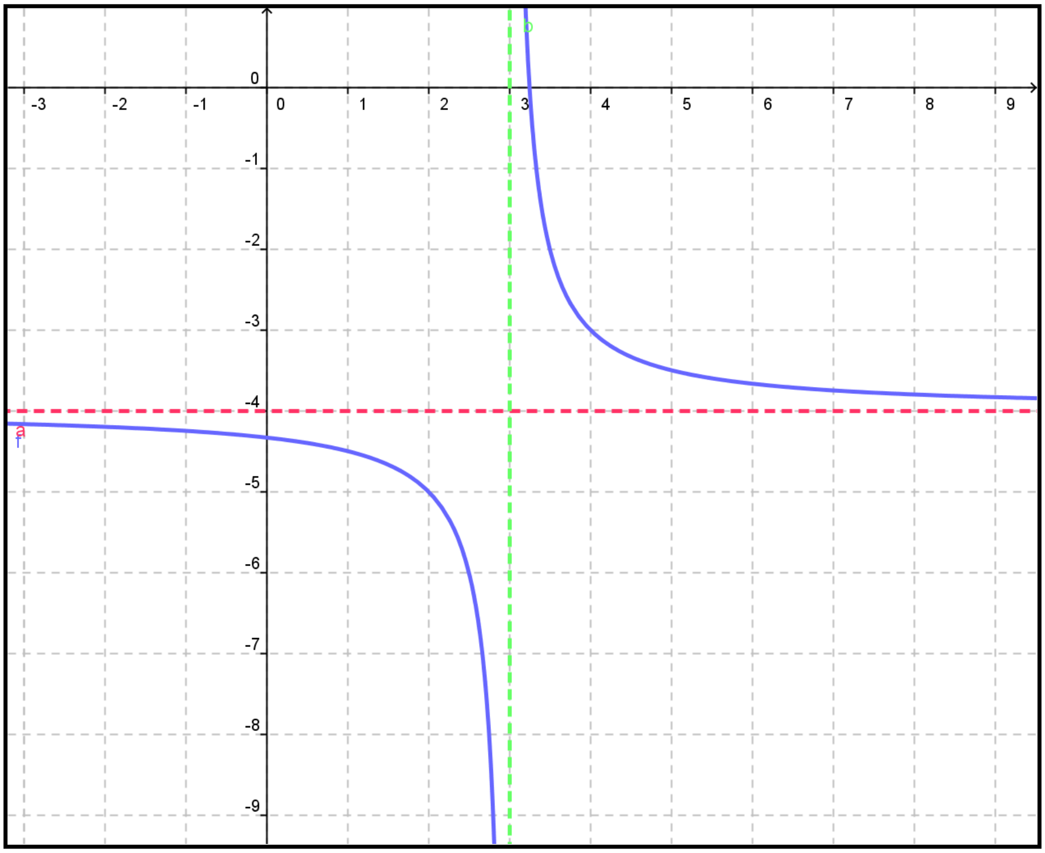
3 14) Consider the function $f(x)=\begin{cases} x^2+kx & x\le 5 \\ 5\sin\left(\frac{\pi}{2}x\right) & x>5 \end{cases}$,

In order for the function to be continuous at x = 5, the value of k must be

2 15) Consider the function $f(x)=\begin{cases} \frac{\sin x}{x} & x\ne 0 \\ k & x=0 \end{cases}$.

In order for the function to be continuous at x = 0, the value of k must be

Use the graph of f(x), shown below, to answer #16-18. (1 pt each).



16) For what value of a is $\lim_{x \to a} f(x)$ nonexistent?

17) $\lim_{x \to \infty} f(x) =$

18) $\lim_{x \to -\infty} f(x) =$

Part III: Designated Deriving! (38 points)

1 1) $$\lim_{h \to 0} \frac{\tan^{-1}(1+h) - \frac{\pi}{4}}{h} =$$

1 2) $$\lim_{h \to 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$$

For #3-8, find the derivative.

2 3) $y = \ln(1+e^x)$

2 4) $y = \csc(1+\sqrt{x})$

4 5) $y = (\tan^2 x)(3\pi x - e^{2x})$

2 6) $y = \sqrt[7]{x^3 - 4x^2}$

3 7) $f(x) = (x+1)e^{3x}$

3 8) $f(x) = \dfrac{e^{x/2}}{\sqrt{x}}$

2 9) Consider the function $f(x) = \sqrt{x-2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?

2 10) If $xy^2 - y^3 = x^2 - 5$, then $\dfrac{dy}{dx} =$

2 11) The distance of a particle from its initial position is given by $s(t) = t - 5 + \dfrac{9}{(t+1)}$, where s is feet and t is minutes. Find the velocity at t = 1 minute in appropriate units.

Use the table below for #12-13.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	4	2	5	½
3	7	-4	$\frac{3}{2}$	-1

2 12) The value of $\dfrac{d}{dx}(f \cdot g)$ at x = 3 is

2 13) The value of $\dfrac{d}{dx}\left(\dfrac{f}{g}\right)$ at x = 1 is

In #14-15, use the table below to find the value of the first derivative of the given functions for the given value of x.

X	$f(x)$	$g(x)$	$f'(x)$	$g('x)$
1	3	2	0	¾
2	7	-4	$\frac{1}{3}$	-1

[2] 14) $[f(x)]^2$ at x = 2 is

[2] 15) $f(g(x))$ at x = 1 is

16) Let f be the function defined by $f(x)=\dfrac{x+\sin x}{\cos x}$ *for* $-\dfrac{\pi}{2}<x<\dfrac{\pi}{2}$.

[2] (a) State whether *f* is an even function or an odd function. Justify your answer..
[2] (b) Find $f'(x)$.
[2] (c) Write an equation for the line tangent to the graph of *f* at the point $(0, f(0))$.

Part IV: Derived and Applied! (27 points)

For #1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

[4] 1) $f(x)=\dfrac{5-4x+4x^2-x^3}{x-2}$

[3] 2) $y=3x^3-2x^2+6x-2$

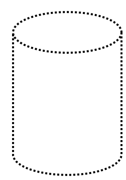
[3] 3) $f'(x)=5x^3-15x+7$

1 4) The graph of the function $y = x^5 - x^2 + \sin x$ changes concavity at x =

3 5) Find the equation of the line tangent to the function $y = \sqrt[4]{x^7}$ at $x = 16$.

1 6) For what value of x is the slope of the tangent line to $y = x^7 + \frac{3}{x}$ undefined?

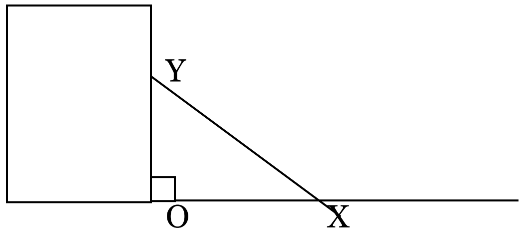
7) *The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of* 261π *cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is* 144π *cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute. (The volume of a cylinder with radius* r *and height* h *is* $\pi r^2 h$, *and the volume of a sphere with radius* r *is* $\frac{4}{3}\pi r^3$.*)*



3 (a) At this instant, what is the height of the cylinder?

3 (b) At this instant, how fast is the height of the cylinder increasing?

8)



A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of ½ foot per second.

[3] (a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.

[3] (b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

Part V: Integral to Your Success! (31 points)

[3] 1) $\displaystyle\int_{-8}^{-1} \frac{x - x^2}{2\sqrt[3]{x}}\,dx$

[3] 2) $\displaystyle\int_{-\pi/6}^{\pi/6} \sec^2 x\,dx$

[1] 3) $\displaystyle\frac{d}{dx}\int_{1}^{x} \sqrt[4]{t}\,dt$

[2] 4) $\displaystyle\frac{d}{dx}\int_{\sin(4x)}^{0} e^t\,dt$

2 5) $\int \frac{x^3}{\sqrt{1+x^4}}dx$

2 6) $\int \frac{\csc^2 x}{\cot^3 x}dx$

2 7) $\int \sqrt{\tan x}\sec^2 x dx$

2 8) What are all the values of *k* for which $\int_2^k x^5 dx = 0$?

3 9) What is the average value of $y = x^3\sqrt{x^4+9}$ on the interval [0, 2]?

2 10) If $\int_a^b g(x)dx = 4a+b$, then $\int_a^b [g(x)+7]dx =$

2 11) The function *f* is continuous on the closed interval [1, 9] and has the values given in the table. Using the subintervals [1, 3], [3, 6], and [6, 9], what is the value of the trapezoidal approximation of $\int_1^9 f(x)dx$?

x	1	3	6	9
f(x)	15	25	40	30

2 12) The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
h(x)	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

13) A particle moves along the x-axis so that, at any time $t \geq 0$, its acceleration is given by $a(t) = 6t + 6$. At time $t = 0$, the velocity of the particle is -9, and its position is -27.

2 (a) Find $v(t)$, the velocity of the particle at any time $t \geq 0$.
1 (b) For what values of $t \geq 0$ is the particle moving to the right?
2 (c) Find $x(t)$, the position of the particle at any time $t \geq 0$.

Part VI: Apply Those Integrals! (26 points)

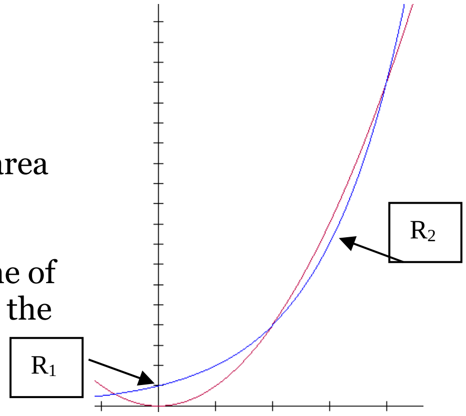
For #1-2, find the general solution to the given differential equation.

3 1) $\dfrac{dy}{dx} = \dfrac{3y}{2+x}$

3 2) $\dfrac{dy}{dx} = y \sin x$

4 3) Find the particular solution to the differential equation $\dfrac{du}{dv} = uv \sin v^2$ if $u(0) = 1$.

4) The shaded regions, R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.



1 (a) Find the x- and y-coordinates of the three points of intersection of the graphs of *f* and *g*.

4 (b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of *f* and *g*. Do not evaluate.

2 (c) Without using absolute value, set up an expression involving one or more integrals that gives the volume of the solid generated by revolving the region R_1 about the line y = 5. Do not evaluate.

5) Let R be the region in the first quadrant under the graph of $y = \frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

3 (a) Find the area of R.

3 (b) If the line x = k divides the region R into two regions of equal area, what is the value of k?

3 (c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.